Number	135630-EN	Topic	Mechanics, rigid bodies, precision measurements			
Version	2016.08.11 / HS	Type	Student exercise	Suggested for grade 12+	p.	1/4

Objective

Determining the local acceleration due to gravity g with great precision.

Principle

The equipment is a model of a so-called **reversion pendulum**. The name refers to the pendulum's ability to be used upside down – there is a pivot at each end. The reversion pendulum is built with different distances from the pivots to the centre of mass, and is subsequently adjusted to have the same oscillation period for the two pivots.

From this, g can easily be calculated.

Equipment

The physical pendulum 218100 consists of a steel rod with a row of holes which are used both as pivots and for attaching weights. The fixed part of the bearing is formed by a knife edge which is attached to stand material or even better clamped to a table edge.

The equipment is provides with 4 disks of steel and 2 of aluminium. These are used in pairs – placed on each side of the steel rod with a bolt. Hereafter, two disks, a bolt and a nut is termed a *weight*.

Two extra sets of nuts and bolts are provided as "trimming weights" and also four washers to be used with the large weights – Note: this is different from exp. 135610-EN.

The reversion pendulum was developed by Henry Kater. The principle was refined by F.W. Bessel and his version can be demonstrated with this equipment (albeit with a lower precision than in the classical precision instruments). The **Bessel pendulum** is a reversion pendulum that with respect to geometrical shape is symmetrical although its mass distribution is asymmetrical. It turns out that this eliminates two error sources caused by the presence of air. (These are buoyancy and the fact that a small amount of air will swing together with the pendulum, causing it to appear heavier.) For details, see the literature list.

Measuring the Period of the Pendulum

Select one of the four different methods given below.

With a stopwatch

You must measure the time for a number of *complete* swings and divide by the number. Precision increases if the watch is started and stopped when the pendulum passes its lowest point where its speed is highest. Use a fixed point behind the pendulum as a reference and don't move your head between start and stop.

Realistically, you cannot hope for uncertainties less than 0.2 s. (If you aim for a 0.5 % precision, the total measuring time must therefore be at least 40 s.)
With the Bessel pendulum, aim for better than 0.1 %; this means measuring times of at least 200 s.

With data logging

Place a motion sensor close by the pendulum, preferably pointing at a weight. (It takes some luck to hit the narrow rod, but it can be done.) Adjust the software to log *position* with a sample rate of 100 Hz. Check that data follows a sine curve reasonably well – large spikes indicate that the sensor misses the target.

Measure for "sufficiently long time" – at least 60 s.

Fit the data with a *damped harmonic oscillation*. Make sure that the fit parameters are shown with sufficient number of digits.

Depending on the software, you get the period T directly, alternatively $\omega = 2\pi/T$.

With a photogate and a timer

Let the pendulum hang motionless. Position the photogate so that the light ray "touches" the edge of the rod – see photo.
Photogate 197550 has a green LED that goes off when the light ray is blocked.

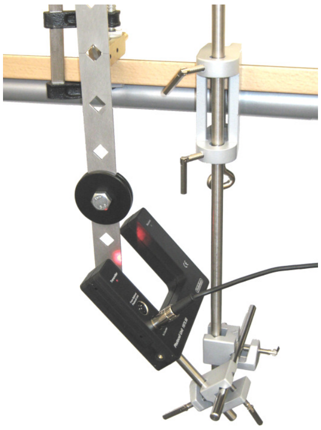
With the pendulum swinging (small amplitude!) the light ray must be blocked for a complete half period and pass through for the other half. This way a period is exactly the time from one blocking to the next.

With **timer 200250** the procedure is like this:

Plug the photogate into DIN socket A.

1. Pull the pendulum a little away from the light ray during the following points
2. Press *Select* until the lamp next to *Period* turns on

3. Wait until the lamp *Continuous* turns on, then press *Memory/Continuous*
4. At last, press *Start/Stop*
5. Now release the pendulum



Results are displayed as the average of two periods – write down. Continue for sufficiently long time. Calculate the mean value.

With SpeedGate

SpeedGate (197570) has two light rays; in this experiment we use the one marked "X".

A status indicator in the display is active when the light ray is blocked. With a motionless pendulum, the "X" light ray should just graze the rod.

Use only small amplitudes: The light ray must be blocked for one half of the period. Select *Period* and *Mean Period* (not *Pendulum Period*).

1. Start the pendulum with a *small* amplitude
2. Press *Reset*
3. Read the mean period when the chosen measuring time expires.

Measuring *g* – Procedure

Place two steel disks with bolt, nut *and washers* in the outermost hole at one end of the rod, and two aluminium disks at the opposite end - *again with washers*. The pendulum is suspended from the next, free hole at each end. The knife edge must be clamped to a table – ordinary stand material is not stable enough. If possible, use a table that is bolted to the wall.

The pivot at the end with the steel weight is called O_1, the other pivot (by the aluminium weight) is called O_2. The corresponding periods are termed T_1 and T_2.

Very precise measurements are required. With a stopwatch, use the average over *at least* 150 periods.

Correctly used, photogates or data logging will give superior results, compared to the stopwatch.

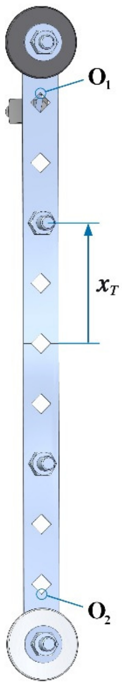
The amplitude must be ***small***. An amplitude of half a centimetre is fine.

The trimming weights, consisting of a bolt and a nut, must always be placed symmetrically around the centre of the rod. Initially, place them in the two holes 50 mm from the centre.

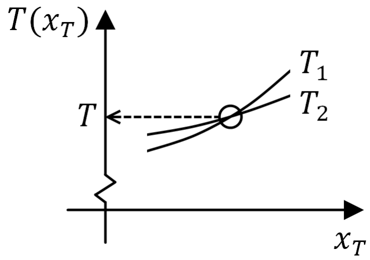
Using pivot O_1, measure T_1. Turn the pendulum around to use O_2 and measure T_2.

Move the trimming weights symmetrically to the next pair of holes and repeat. Do this again with the last pair of free holes.

The periods will vary when the trimming weights are moved. When they are closest to each other T_1 is smaller than T_2. Conversely, when the trimming weights are as far from each other as possible, T_2 is smaller than T_1. For some distance in between, the two periods must be identical. We are seeking the exact value of this common period T:



Use a spreadsheet to plot T_1 and T_2 as a function of the distance x_T from the centre to the trimming weights. Add polynomial trend lines of order 2. Increase the number of vertical subdivisions in order to make it easy to read the common period where the graphs are intersecting.



The final measurement is the distance Δx between O_1 and O_2. It is measured from the upper corner of the top hole to the lowest corner of the bottom hole. With a tape ruler and a magnifying glass it should be possible to do so with a precision of around 0.25 mm. Remove all weights from the rod first.

Now, the local value of the acceleration due to gravity can be found:

$$g = \Delta x \cdot \left(\frac{2\pi}{T}\right)^2$$

The period should be measured with such precision that the Δx measurement determines the total precision limit. On the classical precision instruments, this distance could be measured e.g. by interferometry.

Theory

The **period** of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{Mag}}$$

where I is the moment of inertia with respect to the pivot, M is the total mass of the pendulum, a is the distance between the pivot and the centre of mass, g is the acceleration due to gravity.

Let the moment of inertia for a body with respect to an axis *through the centre of mass* of the body be called I_G . From this, the moment of inertia I, with respect to an arbitrary axis, parallel to the other one, can be found via the **parallel axis theorem** (aka the **Huygens–Steiner theorem**):

$$I = I_G + Ma^2$$

Here, M is the mass of the body in question and a is the distance between the two axes.

This theorem is extremely useful for calculating moments of inertia, except from the most simple cases.

The moment of inertia around resp. O_1 and O_2 are called I_1 resp. I_2 .

The distance between G and O_1 is called x_1 and the distance between G and O_2 is called x_2 .

According to the parallel axis theorem we have that

$$I_1 = I_G + M \cdot x_1^2 \qquad I_2 = I_G + M \cdot x_2^2$$

The two periods of oscillation are given by

$$T_1 = 2\pi \cdot \sqrt{\frac{I_G + M \cdot x_1^2}{M \cdot x_1 \cdot g}} \qquad T_2 = 2\pi \cdot \sqrt{\frac{I_G + M \cdot x_2^2}{M \cdot x_2 \cdot g}}$$

If $T_1 = T_2$, it is seen that

$$\frac{I_G + M \cdot x_1^2}{x_1} = \frac{I_G + M \cdot x_2^2}{x_2}$$

Presuming $x_1 \neq x_2$, this equation can be solved with respect to I_G

$$I_G = M \cdot x_1 \cdot x_2$$

The period is then:

$$T = 2\pi \cdot \sqrt{\frac{x_1 + x_2}{g}} = 2\pi \cdot \sqrt{\frac{\Delta x}{g}}$$

– where Δx designates the distance between the two pivots.

Comparison

The experimentally established value for g can be compared with the theoretical value for the smooth earth ellipsoide (the geoide), corrected for height:

$$\begin{aligned} g = \; & 0{,}0002269\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot \sin^4(\varphi) \\ + \; & 0{,}0516323\,\tfrac{\mathrm{m}}{\mathrm{s}^2} \cdot \sin^2(\varphi) \\ + \; & 9{,}780327\,\tfrac{\mathrm{m}}{\mathrm{s}^2} + C_{FA} + C_B \end{aligned}$$

Here φ is the latitude; the height correction is split into two terms C_{FA} and C_B . (Free Air correction and Bouguer correction). They represent a reduction of g caused by a larger distance to the centre of the earth and an increase in g caused by the mass of the extra layer of soil:

$$C_{FA} = -3{,}086 \cdot 10^{-6}\ \mathrm{s}^{-2} \cdot h$$
$$C_B = 4{,}193 \cdot 10^{-10}\ \mathrm{m}^3\mathrm{s}^{-2}\ \mathrm{kg}^{-1} \cdot \rho \cdot h$$

Here, h is the height above sea level.
The expression for C_B includes ρ , the average density of the soil layers between the position and sea level. Using a typical density of 2670 kg/m^3 , the last two corrections can be written as:

$$C(h) = C_{FA} + C_B = -1{,}966 \cdot 10^{-6}\ \mathrm{s}^{-2} \cdot h$$

In practice, an easy way to find the latitude is on online service like Google Maps. The height over sea level may be found by looking for local geographical data (”GIS”). Or else, a good map will do..

Discussion and evaluation

Make a thorough analysis of all contributions to the experimental uncertainty of g .

Compare the value found for g with the theoretical height-corrected value found for a smooth ellipsoid earth.

Has your measurement possibly demonstrated a local variation in g ?

Literature

D. Candela, K. M. Martini, R. V. Krotkov, and K. H. Langley:
Bessel’s improved Kater pendulum in the teaching lab
American Journal of Physics - June 2001 - Volume 69, Issue 6, pp. 714

Teacher's notes

Concepts used

Centre of mass,
Moment of inertia
– are presumed known

The parallel axis theorem,
Period of oscillation of physical pendulum
– resumed in the text

Reversion pendulum
– the formula for the period is deduced

Mathematical skills

Evaluation of expressions
Plotting of graphs

About the equipment

Treat the Bessel pendulum with caution. The knife edge bearings (including the corners of the square holes) should not be subjected to overload. The corners have a small curvature, which leaves space around the knife. If they get a notch, the bearing cannot rock freely – and it will also be more difficult to measure the distance between the pivots.

(During development of his experiment, a deviation of about 0,4 ‰ was reached, relative to value for the height-corrected geoide.)

A ready-to-use spreadsheet for calculating moments of inertia etc. can be found at www.frederiksen.eu Search for item number 218100.

Detailed equipment list

Specifically for the experiment

218100 Physical pendulum / Bessel-pendulum
001510 Clamp

Larger equipment

Option: Timing with SpeedGate
197570 SpeedGate

Option: Timing with photogate and timer
200250 Universal counter/timer
197550 Photogate

Option: Timing with a datalogger
Motion sensor
Logger or link to PC

Standard lab equipment

(Depending on the timing equipment)

001600 Table clamp
002310 Bosshead, square (1-2 are used)
000850 Retort stand rod 25 cm
000820 Retort stand rod 75 cm
000100 Retort stand Base